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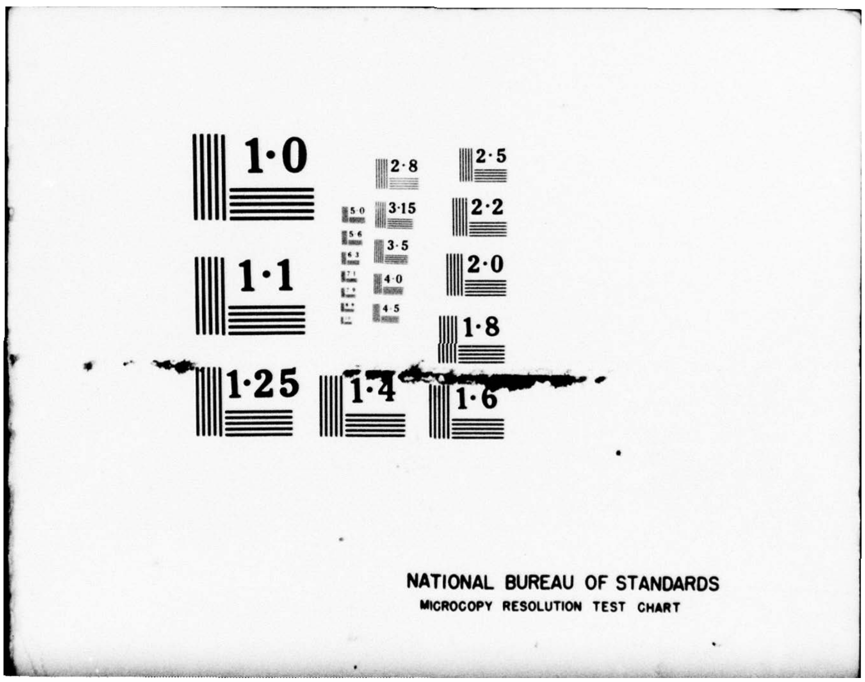
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A SYNTAX-DIRECTED METHOD OF EXTRACTING TOPOLOGICAL REGIONS FROM A SILHOUTTE

by

John O. Amoss

Research supported by
Air Force Office of Scientific Research
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July, 1977

ABSTRACT

The objective of this research is to develop a preprocessing technique for decomposing pictorial data into its topological units and subunits so as to enhance any subsequent feature/primitive extraction. To enhance the later stages of perception implies that each topological unit and its subunits must be labeled in such a way as to reveal their relationship to the other topological units of the pattern. If the labeling can be standardized then the feature/primitive analysis, which is very application-oriented, can process the results of the preprocessor deocmposition as desired. While the proposal for such a decomposition (called segmentation or region analysis) is not new, the concept of decomposition to reveal a pattern's intrinsic structure prior to and independent of feature/primitive analysis is new.

PREFACE

This work was supported by Grant 72-2351 from the Air Force Office of Scientific Research to Professor B. Chandrasekaran of the Department of Computer and Information Science, and conducted at the Computer and Information Science Research Center of The Ohio State University. The Computer and Information Science Research Center of The Ohio State University is an interdisciplinary research organization which consists of the staff, graduate students, and faculty of many University departments and laboratories. This report is based on research accomplished in cooperation with the Department of Computer and Information Science. The research contract was administered and monitored by The Ohio State University Research Foundation.

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1.1 INTRODUCTION

An accepted, generalized model of a pattern recognition system is shown in Figure 1. Classically, the transducer as the input device senses the environment to generate a pattern; the preprocessor readies the pattern for analysis (e.g., noise removal); the extractor analyzes the pattern, extracting information which will enable the classifier to decide to what class of known patterns the input pattern belongs.

There exists both neurophysical evidence in certain animals and psychological evidence in humans that indicate that feature/primitive extraction is part of the recognition process (1). However there also exists strong psychological evidence that the feature/primitive processing alone is insufficient to account for the phenomenon of pattern recognition (2). Such evidence suggests the need for a pre-analytic process to take place before the feature/primitive analysis begins. Such a preprocessor task deals with determining the figure-ground context of a pattern. This task has been referred to by different psychologists as the perception of the figural unity (3), the perception of the generic object (4), and the perception of the topological object (5). This proposal is in contrast to the strictly feature/primitive viewpoint which essentially ignores the context of a pattern, even though such a context has been recognized as an important part of perception (6). However, as pointed out by Deutsch (7), this proposal should not be confused with the Gestalt approach of template matching. The Gestalt viewpoint contends that perception of a figure in a pattern is not a figure in isolation, but a figure set upon a ground (context) and that such perception is the only level of analysis needed, i.e., feature/primitive analysis is not necessary. Thus, it appears, there is justification and need for a preprocessor stage that decomposes a pattern into units which reveal its figure-context relationship. As indicated by Piaget (5) such a decomposition would be topological in nature as opposed to projective-Euclidean in nature. That is, the decomposition should not depend on concepts such as size, angle, distance, parallelism, etc.; but instead on the concepts of adjacency, neighborhood, connectedness, dissection of an object, etc. (8).

As a practical motivation for such a decomposition, consider the following proposal by Breeding (9). Breeding has concluded that while impressive results have been obtained for the automatic identification of aircraft from television patterns using moment invariants (10), such a technique employs global features and is susceptible to the absence of pictorial data. In particular, if a subunit (e.g., wing) of a topological unit (e.g., aircraft) is obstructed because of cloud cover, camouflage, shadows, etc., then misclassification usually results. Thus it would be advantageous to investigate methods of decomposing a topological unit into the subunits which reveal the intrinsic structure of the topological unit. These subunits would then be considered as the input to the feature/primitive extraction stage. Such a decomposition technique should not limit the invariance of the subsequent recognition technique and should be amenable to a braod spectrum of pictorial patterns.

With these ideas in mind, the objective of this research was to develop a preprocessing technique for decomposing pictorial data into its topological units and subunits so as to enhance any subsequent feature/primitive extraction. To enhance the later stages of perception implies that each topological unit and its subunits must be labelled in such a way as to reveal their relationship to the other topological units of the pattern. If the labelling can be standardized then the feature/primitive analysis, which is very application-oriented, can process the results of the preprocessor decomposition as desired. While the proposal for such a decomposition step (called segmentation or region analysis) is not new, the concept of decomposition to reveal a pattern's intrinsic structure prior to and independent of feature/primitive analysis is new.

To accomplish this objective the following questions must be answered:

1) How does one define the topological units of a pattern?
2) How does one label these topological units?
3) How does one define the subunits of a topological unit?
4) How does one label these subunits?

2.1 CONNECT RELATION

What kind of a relation would best define the topological units of a pattern? The topology of a pattern can be represented in many different formalisms, e.g., adjacent cells of a discrete plane, neighboring automata of an array processor, connected vertices of a network grid, etc. To express all these, a generalization of the notion of connectivity is needed. Let X and Y be denumerable sets, then define $\underline{\text{connect}}\langle Y//X\rangle$ as the subset of all elements of X which are adjacent to the elements $\underline{\text{connect}}^0\langle Y//X\rangle = Y \cap X$. If $\underline{\text{connect}}^0\langle Y//X\rangle = \Phi$, then $\underline{\text{connect}}\langle Y//X\rangle = \Phi$. The second argument preceded by a // symbol denotes the restriction of Y to X. Elements adjacent to adjacent elements can be represented by a recursion of connect (see Figure 2).

$$\underline{\text{connect}}^2\langle Y//X\rangle = \underline{\text{connect}}\langle(\underline{\text{connect}}\langle Y//X\rangle)//X\rangle$$

$$\underline{\text{connect}}^3\langle Y//X\rangle = \underline{\text{connect}}\langle(\underline{\text{connect}}\langle(\underline{\text{connect}}\langle Y//X\rangle)//X\rangle)//X\rangle$$

The recursive closure of connect is given by

$$\underline{\hat{\text{connect}}}\langle Y//X\rangle = \underline{\text{connect}}^n\langle Y//X\rangle$$

where n is such that

$$\underline{\text{connect}}^n\langle Y//X\rangle = \underline{\text{connect}}^{n+1}\langle Y//X\rangle.$$

A set X is called a connected set if $\forall x_i \varepsilon X$, $\underline{\hat{\text{connect}}}\langle x_i//X\rangle = X$. Note that since the concept of adjacency is undirected, $\underline{\hat{\text{connect}}}$ is an equivalence relation and thus can partition a set.

3.1 PICTURES

3.1.1 Picture as Input

Physically, an input pattern will be a two-dimensional, binary-valued array. Such an input is practical to obtain when a TV camera is used as the transducer (11). If the TV system is equipped with variable threshold and color variables (12), the real-world environment can be approximated by a set of parameter dependent binary-valued patterns.

Mathematically, an input pattern will be considered as a regular tessellation of the plane into binary-valued cells. For a plane, there are only three possible regular tessellations: triangular, square, and hexagonal. On each such tessellation various indices can be defined such as forward raster, backward raster, diagonal, etc. These indices allow one to talk about the i^{th} cell of a given tessellation. A fixed-size, connected set P of these cells will be called picture cells. This set P will be bounded by a reachable set B called background cells. The values of the background cells do not vary and are a priori. The values of the picture cells do vary and are given by the discrete binary-valued function $\underline{\text{pict}}\langle i\rangle$ where i represents the index of the i^{th} cell for a given tessellation.

3.1.2 Topological Units of a Picture

Using $\underline{\text{pict}}$ as the characteristic function of a set, the following topological subsets of a picture can be defined. The images set I is

$$\{j \;:\; \underline{\text{pict}}\langle j\rangle = b;\; j \in P\}$$

where b may equal either 0 or 1 but not both.

The ground set $\bar{I}$ is defined as the set of all picture cells which are not image cells, i.e., $\bar{I} = P-I$. The surrounds set D is

$$\{j \;:\; B \subset \underline{\hat{\text{connect}}}\langle j//\bar{I}\cup B\rangle;\; j \in \bar{I}\}.$$

Thus D is the subset of all ground cells that are connected to the background B. The holes set H is the subset of all ground cells not connected to the background i.e., $H = \bar{I}-D$. From these definitions it follows that these three topological subsets are pariwise disjoint and thus form a partition of the picture set P, and that for picture $P_a = I_a \cup H_a \cup D_a$.

The basic topological unit of a picture will be a region. A region, R, is any connected subset of a picture. Since a region is a connected set, the following definitions can be made. A cell $j \in R$ is a border cell if $\underline{\text{connect}}\langle j//P\cup B\rangle \not\subseteq R$, i.e., not all the cells adjacent to j are in R (see Figure 3). The border cells of a region are partitioned into contours ${}_jC = \hat{\text{connect}}\langle j//X\rangle$ where $j \in X = \{\text{border cells}\}$. Thus a contour of a region is a connected set of border cells, ${}_jC \subseteq R$. A simply-connected region is a region that has only one contour (see Figure 4). A multiply-connected region is a region that has two or more contours (see Figure 5).

The sets I, D, and H can be partitioned into topological regions called respectively, images, surrounds, and holes. Each image ${}_iI = \underline{\hat{\text{connect}}}\langle i//I\rangle$; each surround ${}_jD = \underline{\hat{\text{connect}}}\langle j//D\rangle$; and each hole ${}_kH = \underline{\hat{\text{connect}}}\langle k//H\rangle$ (see Figure 6).

From these topological regions a pseudo region, called a collage, can be defined. The collage set K is the set of all collages. A collage is fabricated by combining topological regions and is useful for decomposing regions nested within regions (see Figure 7). A collage may or may not be simply-connected.

3.1.3 Print of a Picture

If one agrees to talk about picture subsets only, then P can be considered as the universal set. As a result, one can agree that $i \notin X$ implies $i \in P-X$ since $X \subseteq P$. Define the print of a picture subset X, denoted ${}_X P$, as

$$\underline{\text{pict}}_X\langle i\rangle = \begin{cases} 1 & i \in X \\ 0 & i \notin X. \end{cases}$$

This leading subscript notation is used to differentiate prints from the usual following subscript notation for subsets. The print of a single cell j, denoted ${}_j P$, is just

$$\underline{\text{pict}}_j\langle i\rangle = \begin{cases} 1 & i = j \\ 0 & i \neq j. \end{cases}$$

The relationship between a print and a picture is given by the develop relation.

$${}_I P = \underline{\text{develop}}\langle P\rangle.$$

That is, develop operates on the binary values of a picture such that the image cells are labelled 1 and the ground cells are labelled 0. To do this labelling it is necessary to have a priori knowledge about how the TV system assigns binary values. The importance of the develop relation lies in the fact that one can work with prints knowing that they can always be found for any picture.

Because pictures are fixed-size sets it makes no sense to talk about P_a being a subset of P_b. However, two pictures are said to be equal if ${}_{I_a}P = {}_{I_b}P$, and not equal if ${}_{I_a}P \neq {}_{I_b}P$.

3.1.4 Print Operations

For a given tessellation and a given index one can define a print operation between the cells of one print and the cells of another print to produce a new print. For example, print operations analogous to Boolean operations can be defined since each cell takes on only two values. Define

$$\underline{\text{and}}\langle {}_X P, {}_Y P\rangle = {}_Z P$$

where $Z = \{j \; ; \; \underline{\text{pict}}_X\langle j\rangle \; \& \; \underline{\text{pict}}_Y\langle j\rangle = 1\}$ and & is the Boolean and.

Define

$$\underline{\text{or}}\langle {}_X P, {}_Y P\rangle = {}_Z P$$

where $Y = \{j \; : \; \underline{\text{pict}}_X\langle j\rangle \mid \underline{\text{pict}}_Y\langle j\rangle = 1\}$ and $\mid$ is the Boolean or.

Define

$$\underline{\text{comp}}\langle {}_{X}P\rangle = {}_{Y}P$$

where $Y = \{j \;:\; \text{pict}_{X}\langle j\rangle = 0\}$. Also a generalized picture <u>and</u> and <u>or</u> can be defined.

$$\prod_{i}^{n} {}_{X_i}P = \underline{\text{and}}\langle {}_{X_n}P, \underline{\text{and}}\langle {}_{X_{n-1}}P, \ldots, \underline{\text{and}}\langle {}_{X_2}P, {}_{X_1}P\rangle \ldots\rangle\rangle, \text{ also } \prod_{j}^{m} {}_{j^X}P, \quad \prod_{i,j}^{m,n} {}_{j^{X_i}}P$$

$$\Gamma_{i}^{n} {}_{X_i}P = \underline{\text{or}}\langle {}_{X_n}P, \underline{\text{or}}\langle {}_{X_{n-1}}P, \ldots, \underline{\text{or}}\langle {}_{X_2}P, {}_{X_1}P\rangle \ldots\rangle\rangle, \text{ also } \Gamma_{j}^{m} {}_{j^X}P, \quad \Gamma_{i,j}^{m,n} {}_{j^{X_i}}P$$

Likewise, one can define a picture operation on the cells of an existing picture to produce a new picture. Define

$$\underline{\text{index}}\langle {}_{X}P\rangle = {}_{j}P$$

where j is the first index $j \varepsilon X$. Define

$$\underline{\widehat{\text{reach}}}\langle {}_{Y}P // {}_{X}P\rangle = {}_{Z}P$$

where $Z = \underline{\text{connect}}\langle Y//X\rangle$. Similar to before, $\underline{\widehat{\text{reach}}}^{0}\langle {}_{Y}P//{}_{X}P\rangle = {}_{Z}P$ where $Z = X \cap Y$, in other words $\underline{\widehat{\text{reach}}}^{0}\langle {}_{Y}P//{}_{X}P\rangle = \underline{\text{and}}\langle {}_{Y}P, {}_{X}P\rangle$. Define

$$\underline{\text{remain}}\langle {}_{Y}P//{}_{X}P\rangle = \underline{\text{and}}\langle {}_{X}P, \underline{\text{comp}}\langle \underline{\widehat{\text{reach}}}\langle {}_{Y}P//{}_{X}P\rangle\rangle\rangle.$$

The <u>remain</u> operation can be considered as a complement to the <u>reach</u> operation. That is, while the <u>reach</u> operation produces a print of all cells extracted from an existing print, the <u>remain</u> operation produces a print of all cells remaining in the existing print after a <u>reach</u> extraction (see Figure 8).

To show how these picture operations can be used, consider the recurring problem of extracting the next region with index greater than k from the subset X. Assume the next such region has index j, then in terms of prints

$$\underline{\widehat{\text{reach}}}\langle \underline{\text{index}}\langle \underline{\text{remain}}\langle \Gamma_{i}^{k} {}_{i}P//{}_{X}P\rangle\rangle//{}_{X}P\rangle.$$

$\underline{\text{remain}}\langle \Gamma_{i}^{k} {}_{i}P//{}_{X}P$ will produce a print ${}_{Z}P$ that has all regions of index k or less removed from it. The $\underline{\text{index}}\langle {}_{Z}P\rangle$ must then be ${}_{j}P$ by assumption, resulting in $\underline{\widehat{\text{reach}}}\langle {}_{j}P//{}_{X}P\rangle$, the desired result.

4.1 SUBREGION RELATION

What kind of a relationship would best reveal the intrinsic structure of a pattern? Since the topological units and subunits are being generated by a decomposition, a hierarchical type relation is implied. That is to say, a decomposition implies that the topological units and subunits are related such that they recursively fit together to build up the whole pattern. Thus it is assumed that the choice of a hierarchical relation (partial order relation (14)) for labelling the topological units will best reveal the intrinsic structure of the pattern and hence enhance any subsequent perception processing.

Since a region has been chosen as the basic topological unit (as opposed to a subset), a subregion relation was chosen as the hierarchical relation. This partial order relation was defined as ${}_iR \leq {}_jR$ if ${}_iR \subseteq {}_jR$ and ${}_iR < {}_jR$ if ${}_iR \subset {}_jR$. While conceptually simple, this relation has proven effective for two reasons. First, because of the way topological regions of a picture are spatially represented, if ${}_iK \cap {}_jR \neq \Phi$, then either ${}_jR < {}_iK$ or ${}_jR = {}_iK$. Second, because the subregion relation can be applied to region decomposition also, a unified theory can be presented for topological decomposition.

5.1 PICTURE DECOMPOSITION

A picture in its simplest form has only a multiply-connected surround and a simply-connected image. It is the presence of a multiply-connected image that introduces holes and nested images. However such multiply-connected regions can be changed to a pseudo simply-connected region by creating a collage. Such a collage would consist of the multiply-connected region plus all other topological regions "inside" the multiply-connected region. Thus any picture can be decomposed into its topological regions (${}_iI$, ${}_jD$, ${}_kH$) by representing it as a sequence of collages. Each collage will be decomposed into its "outer" topological region while the "inner" topological regions will form a new collage. The decomposition stops when the new collage is a single topological region, and hence can not be segmented anymore.

Define the $\underline{\text{surround}}<{}_XP>$ as

$$\{i\ :\ B\subset \underline{\hat{\text{reach}}}\ i//(P-X)\cup B>:\ i\varepsilon P-X\}.$$

Now given the print ${}_IP$ of any picture the following prints can be derived

(1) $\quad {}_DP = \underline{\text{surround}}<{}_IP>$

(2) $\quad {}_{{}_jD}P = \underline{\hat{\text{reach}}}<\underline{\text{index}}<\underline{\text{remain}}<\bigcap_{i}^{k}\ {}_iP//{}_DP>>//{}_DP>$

(3) $\quad {}_KP = \underline{\text{comp}}<\underline{\text{surround}}<{}_IP>>.$

Each ${}_jD$ represents a surround and K represents the collages set. Some of the collages of ${}_KP$ represent simply-connected images, and others represent mutliply-connected images with holes and other images "inside" them. The mutliply-connected images are extracted as part of a collage.

(4) $\quad {}_{{}_jK}P = \underline{\hat{\text{reach}}}<\underline{\text{index}}<\underline{\text{remain}}<\bigcap_{i}^{k}\ {}_iP//{}_KP>>//{}_KP>$

and then segmented by

(5) $\quad {}_{{}_jI}P = \underline{\text{and}}<\underline{\text{surround}}<\underline{\text{and}}<\underline{\text{comp}}<{}_IP>,\ {}_{{}_jK}P>>,\ {}_{{}_jK}P>.$

A new collage can now be derived from each exisitng collage minus the multiply-connected image by

(6) $\quad {}_KP = \underline{\text{and}}<\underline{\text{comp}}<\underline{\text{surround}}<\underline{\text{and}}<\underline{\text{comp}}<{}_IP>,\ {}_{{}_jK}P>>>,\ {}_{{}_jK}P>.$

Some of the collages of (6) represent simply-connected holes and others represent multiply-connected holes with images and other holes "inside" them. The multiply-connected holes are extracted as part of a collage as in (4) and then segmented by

(7) $\quad {}_{{}_jH}P = \underline{\text{and}}<\underline{\text{surround}}<\underline{\text{and}}<{}_IP,\ {}_{{}_jK}P>>,\ {}_{{}_jK}P>.$

A new collage can now be derived from each existing collage minus the multiply-connected hole by

(8) $\quad {}_K P = \underline{\text{and}}<\underline{\text{comp}}<\underline{\text{surround}}<\underline{\text{and}}<{}_I P,\ {}_{n^K} P>>>,\ {}_{n^K} P>.$

Some of the collages of (8) represent simply-connected images and others, multiply-connected images with holes and other images "inside" them. These can be decomposed using (4) and (5) and so forth.

5.1.1 Grammar Model

The picture decomposition can be modelled by a context-free phrase structured grammar.

$$V_N = \{P, {}_jD, D, {}_jK, K, {}_jI, I, {}_jH, H, R\},$$
$$V_T = \{d,\ ?,\ .,\ :,\ ;,\ ),\ (\},$$

and the production rules are:

$$P \rightarrow (DK)?$$
$$P \rightarrow ?$$
$$D \rightarrow {}_jDD$$
$$D \rightarrow {}_jD$$
$$K \rightarrow {}_jKK$$
$$K \rightarrow {}_jK$$
$${}_jK \rightarrow ({}_jIK)$$
$${}_jK \rightarrow {}_jI$$
$${}_jK \rightarrow ({}_jHK)$$
$${}_jK \rightarrow {}_jH$$
$${}_jD \rightarrow R:$$
$${}_jI \rightarrow R.$$
$${}_jH \rightarrow R;$$
$$R \rightarrow d$$

where P is the start symbol and d represents some kind of descriptor for a region. Note how the punctuation marks label the regions as being either a surround, an image, or a hole. The string derivation for Figure 2 is:

$$P \quad (DK)?$$
$$\overset{*}{\Rightarrow} ({}_aD\,{}_cD({}_bI\,{}_dH({}_fH\,{}_gI))\,{}_eI)?$$

(9) $\quad \overset{*}{\Rightarrow} (d{:}d{:}(d.d;(d;d.))d.)?$

Because the grammar is context-free there exists a derivation tree for expression (9). Such a tree depicts the hierarchical relationship between the topological regions of the picture. That is to say, from such a tree, or more precisely, from a string of the language, a Hasse diagram for the subregion relation can be derived. For example, if the parentheses are interpreted as levels in the Hasse diagram and the descriptors/punctuation marks as topological region labels, then the Hasse diagram of Figure 9 can be derived for string (9).

Because of this grammar model, the decomposition can be defined as the output of a parser. The parser proceeds top-down when it is given a picture and produces as output a string of topological region descriptors. The parser proceeds bottom-up when it is given a string of topological region descriptors and produces as output a picture. In each case the output is uniquely determined to within an index only, e.g., compare Figure 10 with Figure 11.

6.1 REGION DECOMPOSITION

The principles of picture decomposition can be applied to region decomposition if a region could be separated or disconnected. This separation would create new regions which could then be hierarchically related through the subregion relation. Intuitively, one could envision a simply-connected region being decomposed and related as illustrated in Figure 12. Basically, cells or areas of decomposition are determined and the region is separated at these, thus forming new regions. Such an operation will be referred to as forming the separable subregions of a region. These separable subregions, denoted ${}_jR$, will be considered as the topological subunits of a region.

6.1.1 Skeleton Graph

Let S represent the medial axis or internal skeleton of a region R as defined by Blum (14). Such a descriptor is located at the median, between the contours of a region and forms a connected graph-like region (see Figure 13). The skeleton of a region can be generated by successively removing certain contour cells (15). Let $\underline{skel}\langle {}_RP//_{M_i}P\rangle$ denote the picture operation that executes one removal of contour cells from region R restricted by mask M_i. Then the medial axis for region R restricted by mask M_i is given by the recursive closure of the $\underline{skel}$ operation,

$${}_SP = \underline{\hat{skel}}\langle {}_RP//_{M_i}P\rangle.$$

The skeleton is made up of branches and nodes N, the places where three or more branches meet. The skeleton will be represented by a graph G = (V,E). The vertices $V = \{v_1, v_2, \ldots, v_n\}$ are the end cells of the branches and the nodes N. The edges $E = \{e_1, e_2, \ldots, e_m\}$ are the branches and each is associated with two vertices. The skeleton graph will be assumed to have no self-loops or parallel edges, so that each branch of the skeleton can be uniquely associated with an edge. The skeleton graph for a simply-connected region has no circuits and hence is a tree.

6.1.2 Separable Nodes

Define two special prints: the universal print ${}_UP$, $\underline{pict}_U\langle i\rangle = 1\ \forall i \varepsilon P$; and the empty print ${}_\Phi P$, $\underline{pict}_\Phi\langle i\rangle = 0\ \forall i\varepsilon P$. Define the picture operation of growing a skeleton cell j as

$$\underline{grow}\langle {}_jP//_RP\rangle = \underline{reach}^n\langle {}_jP//_UP\rangle$$

where n is the minimum y such that

$$\underline{and}\langle {}_RP,\ \underline{reach}^y\langle {}_jP//_UP\rangle\rangle \neq \underline{or}\ \langle {}_RP,\ reach^y\ \langle {}_jP//_UP\rangle\rangle.$$

The growth of a set of skeleton cells X, is defined as

$$\underline{growth}\langle {}_XP//_RP\rangle = \underset{i}{\Gamma}\ \underline{grow}\langle \underline{and}\langle {}_iP, {}_XP\rangle//_RP\rangle.$$

Two nodes n_i, n_j $N \subseteq S$ of a region R are nonseparable if

$$\underline{and}\langle \underline{grow}\langle {}_{n_i}P//_RP\rangle,\ grow\langle {}_{n_j}P//_RP\rangle\rangle \neq {}_\Phi P,$$

i.e., their growths merge. Generalizing this, the nodes $N \subseteq S$ of a region R can be partitioned by the separable relation

$$N_i = \underline{sep}\langle n_i, N//R\rangle = \{n_j \;:\; n_i \text{ and } n_j \text{ are nonseparable}\}.$$

Thus all the nodes in a block N_i of the partition of N are nonseparable, but as a group they create a set of nodes separable from all other nodes of N. Each N_i is called a separable node.

6.1.3 Separable Subregions

Let $N_i \subseteq N \subseteq S$ for some region R, then the separable subregions adjacent to N_i are given by the print

$$_{Z}P = \underline{and}\langle {}_{R}P,\ \underline{comp}\langle \underline{growth}\langle_{N_i} P//_{R}P\rangle\rangle\rangle.$$

A print of a separable subregion $_jR$ can be obtained in the usual fashion from $_ZP$ by

$$_{_jR}P = \hat{reach}\langle index\langle remain\langle \Gamma_i^k\ {}_iP//_{Z}P\rangle\rangle//_{Z}P\rangle.$$

Let the part of skeleton associated with separable subregion $_jR$ be denoted $_jS$, then a print of $_jS$ can be obtained by

$$_{_jS}P = and\langle {}_SP, {}_{_jR}P\rangle.$$

6.1.4 Primitive Separable Subregion

Each subregion $_jR$ can be further decomposed into separable subregions by selecting some separable node $N_i \subseteq {}_jS$ and growing it. If an $_jS$ has no nodes, further decomposition of the subregion is not possible. This lack of nodes provides a stopping criterion for the decomposition. Define a primitive separable subset as a subregion $_jR$ whose associated skeleton $_jS$ has no nodes. Obviously if skeleton S of region R has no nodes, then R itself is a primitive separable subregion. Figure 14 shows a region and its skelton that has four nodes, and three separable nodes. The center separable nodes has four separable subregions adjacent to it, the other two separable nodes have three separable subregions adjacent to them. There are a total of eight primitive separable subregions.

6.1.5 Shared Separable Subregion

Figure 14 shows a decomposition where the separable nodes were selected one at a time or one-selected; however, the separable nodes can also be multiply-selected. For mutliply-selected separable nodes, some of the separable subregions will be adjacent to more than one separable node. Such subregions are called shared separable subregions (see Figure 15).

6.1.6 Redundancy

The effect that these shared subregions have on the results of the decomposition is to appear as a redundant subregion in the hierarchical relationship. If a region decomposition contains no shared separable subregions it is called nonredundant; otherwise, it is called redundant. A redundant region decomposition in which all shared separable subregions are primitive is called primitive redundant.

6.1.7 Grammar Model

The region decomposition can be modelled by a context-free phrase structured grammar.

$$V_N = \{R,N,O,E,X\},$$
$$V_T = \{e,n,[,]\},$$

and the production rules are:

$$
\begin{aligned}
R &\rightarrow [O] \\
R &\rightarrow [N] \\
R &\rightarrow [N_i] \\
R &\rightarrow [E] \\
X &\rightarrow [O] \\
X &\rightarrow [O]X \\
O &\rightarrow [N]O \\
O &\rightarrow [N] \\
X &\rightarrow [N]X \\
X &\rightarrow [N] \\
N &\rightarrow N_i X \\
N_i &\rightarrow n \\
X &\rightarrow [E]X \\
X &\rightarrow [E] \\
E &\rightarrow e \quad .
\end{aligned}
$$

R is the start symbol; while e represents a descriptor for an edge, i.e., a primitive separable subregion, and n represents a descriptor for a separable node. The symbol 0 stands for a pseudo node. A pseudo node is a hypothetical node that enables mutliply-selected separable nodes to have a common ancestor. The string derivation for Figure 10 is:

$$
\begin{aligned}
&R \quad [O] \\
&\overset{*}{\Rightarrow} [[N_i[E][E][N_i[E][E][E]]][N_i[E][E][N_i[E][E][E]]]] \\
(10) \quad &\overset{*}{\Rightarrow} [[n_1[e_1][e_2][n_3[e_3][e_4][e_6]]][n_2[e_5][e_2][n_3[e_3][e_4][e_6]]]].
\end{aligned}
$$

Because the grammar is context-free there exists a derivation tree for string (10) which depicts the hierarchical relationship between the separable subregions of the region. For example, if the brackets are interpreted as levels in the Hasse diagram, and the edge descriptors as separable subregion labels, then the Hasse diagram of Figure 16 can be derived for string (10).

Also because of this grammar model, the decomposition can be defined as the output of a parser, just as was done for the picture decomposition. Once again the output of the parser is uniquely determined to within an index only.

7.1 VERTEX ATTRIBUTES

The hierarchical relationship for the separable subregions of a region is completely dependent on the order in which the separable nodes of the skeleton graph are selected, and whether they are one- or mutliply-selected. To guide the selection of separable nodes, the different characteristics of a skeleton graph must be sorted out so that the cause and effect relationship between vertices and edges can be identified.

The syntax of a graph characterizes the interconnection of the vertices and edges. The semantics characterizes the size and shape of the edges and vertices. A graph with the same syntax can have infinitely many semantic characterizations. The semantics of a particular graph can be defined by associating attributes with each edge and vertex. The attributes can be thought of as functions whose values are assigned depending on the meaning that the vertex or edge has for a particular graph.

For selecting nodes, the vertex attributes will be used. Two types of vertex attributes have been recognized as aids in giving a "meaningful" ordering for the nodes of a skeleton: cluster and selection. A cluster attribute is one that characterizes a local property of a vertex. The concept of vertex clustering is based on pattern recognition work which is interested in grouping points using a local optimization criterion (16). A selection attribute is one that characterizes a global property of a vertex. The concept of vertex selection is based on operational research work which is interested in choosing points using a global optimization criterion (17).

7.1.1 Cluster Attributes

For a vertex v_i define the following useful cluster attributes:

1) $\underline{\text{degree}}\langle v_i \rangle = n$ where n is the number of edges incident on v_i;
2) $\underline{\text{size}}\langle v_i \rangle = s_i$ where $\underline{\text{grow}}\langle_{v_i} P//_R P\rangle = \underline{\text{reach}}^{s_i}\langle_{v_i} P//_U P\rangle$;
3) $\underline{\text{edge}}\langle v_i \rangle = \{m_{ij}, m_{ik}, \ldots, m_{in}\}$ where $v_j, v_k, \ldots, v_n$ are all the vertices adjacent to v_i and $m_{ij} = \underline{\text{metric}}\langle v_i, v_j \rangle$.

Given a cluster attribute, a cluster relation can be defined. A cluster relation for the a_i attribute on a connected graph (V,E) about vertex v_k, denoted $\underline{a_i\text{-cluster}}\langle v_k, (V,E)\rangle$, is defined as the set of all connected vertices from vertex v_k that have an equivalent a_i attribute value.

Because the cluster relation is an equivalence relation, each a_i attribute forms a partition of the vertices V. Denote such a partition as Φ_{a_i}. A set of such partitions $\{\Phi_{a_1}, \Phi_{a_2}, \ldots, \Phi_{a_n}\}$ forms a partial ordered set under the refinement relation (18). Thus the logical combination (conjunction and disjunction) of cluster attributes can be defined from the greatest lower bound and the least upper bound operations on this partial ordered set. For example, a partition of all vertices with equal degree and equal size can be formed from the least upper bound of partitions Φ_{degree} and Φ_{size}.

7.1.2 Selection Attributes

For a vertex v_i, define the following useful selection attributes:

1) $\underline{\text{eccen}}\langle v_i \rangle = \underline{\max}\langle s_j * m_{ij} \rangle$;
2) $\underline{\text{weight}}\langle v_i \rangle = \Sigma s_j * m_{ij}$;

where $1 \le j \le \underline{\text{cd}}\langle V \rangle$ and $\underline{\text{cd}}$ is the cardinality of the set.

Given a selection attribute, a select relation can be defined. A select relation for the a_i attribute on a connected graph (V,E), denoted $\underline{a_i\text{-select}}\langle (V,E) \rangle$, is defined as the set of all vertices that have the minimal a_i attribute value.

For the special case when all s_i are equal, the minimum eccentricity corresponds to the center of the graph and the minimum weight corresponds to the median of the graph (19). The definitions as given here are dubbed the weighted center and the weighted median, respectively.

7.1.3 Simply-Connected Regions

Insight into the relationship between separable nodes, separable subregions, and vertex attributes can be gained through the following theorems. Denote N_s as the set of nodes selected by a select relation; N_{cs} as the set of nodes generated by forming clusters about each node of N_s; and an n-selected separable node as either a one- or a multiply-selected separable node.

Lemma 1:

If the set N_{cs} is connected, then the set of all n-selected separable nodes generated from N_{cs} is connected.

Lemma 2:

If the set N_s is connected, then the set N_{cs} is connected.

Theorem 1:

If each N_s of a redundant decomposition of a simply-connected region is connected, then the decomposition is primitive redundant.

Lemma 3:

If two weighted centers (weighted medians) of a tree T = (V,E) are non-adjacent, then all the vertices in the path connecting the two weighted centers (weighted medians) cannot be weighted centers (weighted medians).

Lemma 4:

If two weighted centers (weighted medians) of a tree T = (V,E) are nonadjacent, then at least one vertex in the path connecting the two weighted centers (weighted medians) must be a weighted center (weighted median).

Theorem 2:

Every tree T = (V,E) has only one or two weighted centers (weighted medians) and the two, if they exist, must be adjacent, hence connected.

Theorem 3:

If the weighted center or weighted median is used as a selection attribute, any redundant decomposition of a simply-connected region will be primitive redundant.

8.1 IMPLEMENTATION

The picture operations were implemented on a binary array processor as proposed by Breeding (9). For this work, the proposed hardware array processor was simulated by the software system dubbed GAPS (General Array Processor Simulator). Besides the picture operations already mentioned, operations to smooth contours, fill gaps, and remove salt-and-pepper noise were also defined. In addition, tasks which use the array processor and conventional arithmetic computation were defined to prune the branches of the noisy skeleton, and to screen out regions which were considered too small to be topologically relevant.

For the special case of a simply-connected region when no cluster relations are defined and the selection attribute is the center (i.e., all s_i are assumed equal), the decomposition can be completely implemented by picture operations. Such picture operations were used on the aircraft decomposition suggested by Breeding. A print of an aircraft was first parsed in a top-down fashion. The resulting string description was then analyzed to extract and label the separable subregions corresponding to the wings, the nose section, and the tail section.

9.1 SUMMARY AND CONCLUSIONS

In the past, the decomposition of patterns has been viewed as two apparently distinct steps: 1) decomposition into topological units; 2) decomposition of each unit into "primitives". Step 1 was realized either by the classical method of contour following (22), or by the more recent development of heuristic analysis of the contours (23,24). Step 2 was realized either by restricting the units to line drawings (25,26), by approximating the units' contours by straight lines (27,28), or by approximating the units by convex polygons (29-32).

In contrast, the decomposition described here considers these two steps to be related. The pattern was first considered as a fixed-size set of picture cells, then its topological units (regions) were defined and hierarchically related. These concepts were then extended by defining topological subunits (separable subregions) of a region. These subregions were then related using the same hierarchical relation as before.

In comparison, the picture decomposition described here is conceptually similar to Brice and Fennema's region analysis (24). Brice and Fennema deal with general gray level pictures, and approximate the region contours by straight lines so heuristic analysis can be used to label the regions. In contrast this research applies to binary-valued pictures only, does not approximate the contours, and uses a nonheuristic method of labelling the regions.

In comparison, the region decomposition described here is conceptually similar to Pavlidis formation of convex subsets (29,30). Pavlidis deals with regions approximated by polygons, and generates a set of overlapping polygons called primary subsets whose union gives the polygon region (see Figure 17). The generation of the region's half-plane extensions is necessary to locate the primary subsets. The areas where the primary subsets overlap are called the nuclei. A graph where the nuclei and primary subsets are vertices, and the edges denote an intersection relationship, can be derived to represent the region. This graph is not dependent on the order in which the nuclei of the region are determined. The graph is a web and the decomposition of the region it represents can be modelled by a context-sensitive web grammar. In contrast this research does not approximate the contours, and generates a set of primitive separable subregion whose union does not give the region. The generation of the region's skeleton is necessary to locate the primitive separable subregions. A graph where the separable nodes and primitive separable subregions are vertices, and the edges denote a hierarchical relationship, can be derived to represent the region. This graph is dependent on the order in which the separable nodes are selected. The graph is a tree and the decomposition which it represents can be modelled by a context-free phrase structured grammar. Interestingly enough, a separable node always corresponds to a nuclei; however, not every nuclei corresponds to a separable node, e.g., compare Figures 17 and 18 to Figure 19.

Ignoring for the moment the appropriateness of using a hierarchical relation for labelling regions and subregions, two conclusions about this research can be made. First, the fact that the picture decomposition is topologically-oriented and can be implemented by local parallel operations, indicates it may be mimicking certain human-perception tasks suggested by psychologists (2). Second, the fact that the region decomposition can invariantly extract the protrusions (separable subregions) of an object indicates that it may be a valuable tool for defining the elusive "primitives" needed for syntactic recognition (33). In addition, because the formation of separable subregions can be implemented by local parallel operations, it may be mimicking certain feature/primitive extraction capabilities observed in humans.

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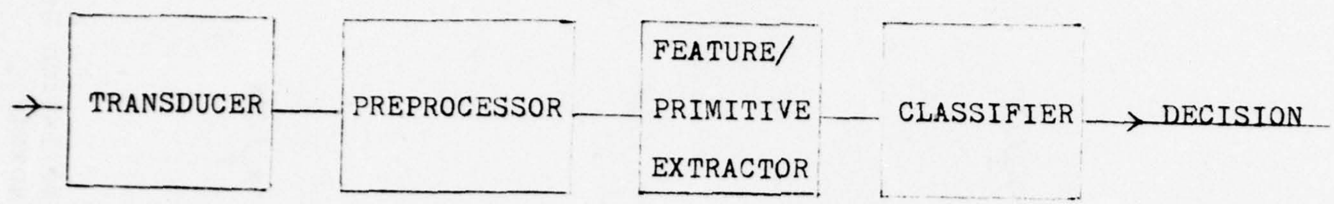
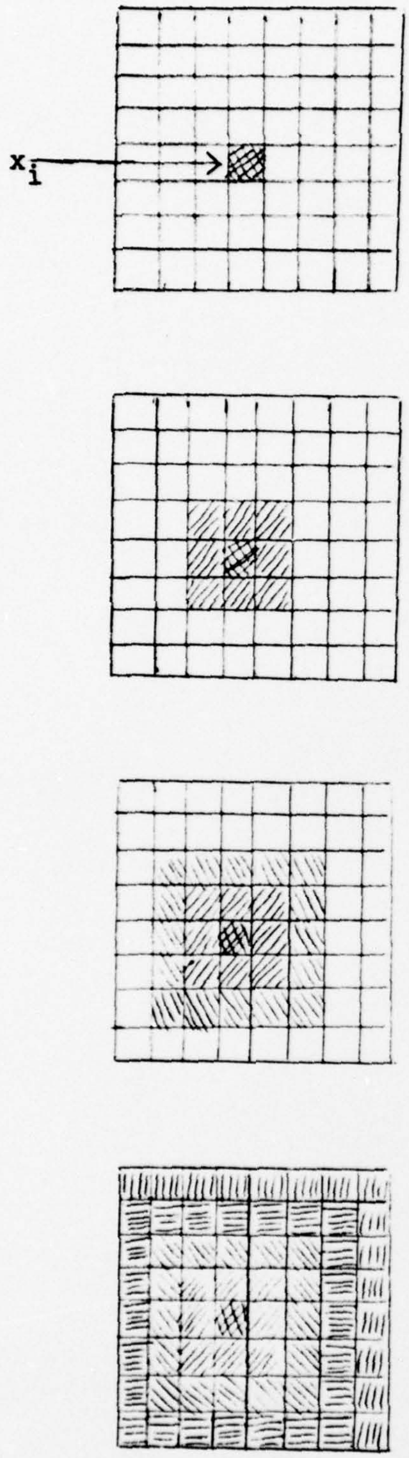


Figure 1. Generalized Pattern Recognition Model



$\underline{\text{connect}}^0\langle x_i//X\rangle$

$\underline{\text{connect}}\ \langle x_i//X\rangle$

$\underline{\text{connect}}^2\langle x_i//X\rangle$

$\underline{\widehat{\text{connect}}}\ \langle x_i//X\rangle$

Figure 2. <u>connect</u> Relation on a Square Tessellation Using Moore Neighborhood to Define Adjacency

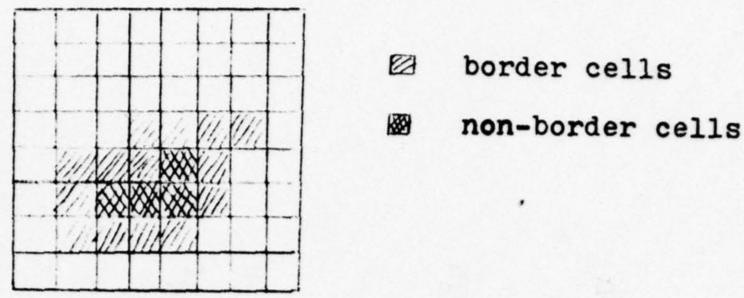


Figure 3. Region: Border Cells and Non-border Cells

Figure 4. Simply-Connected Region and Contour

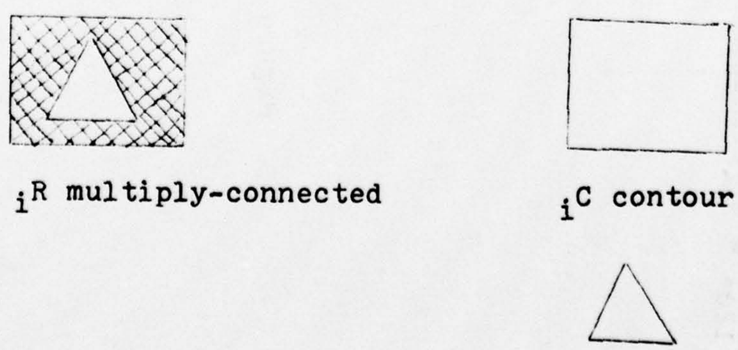


Figure 5. Multiply-Connected Region and Contours

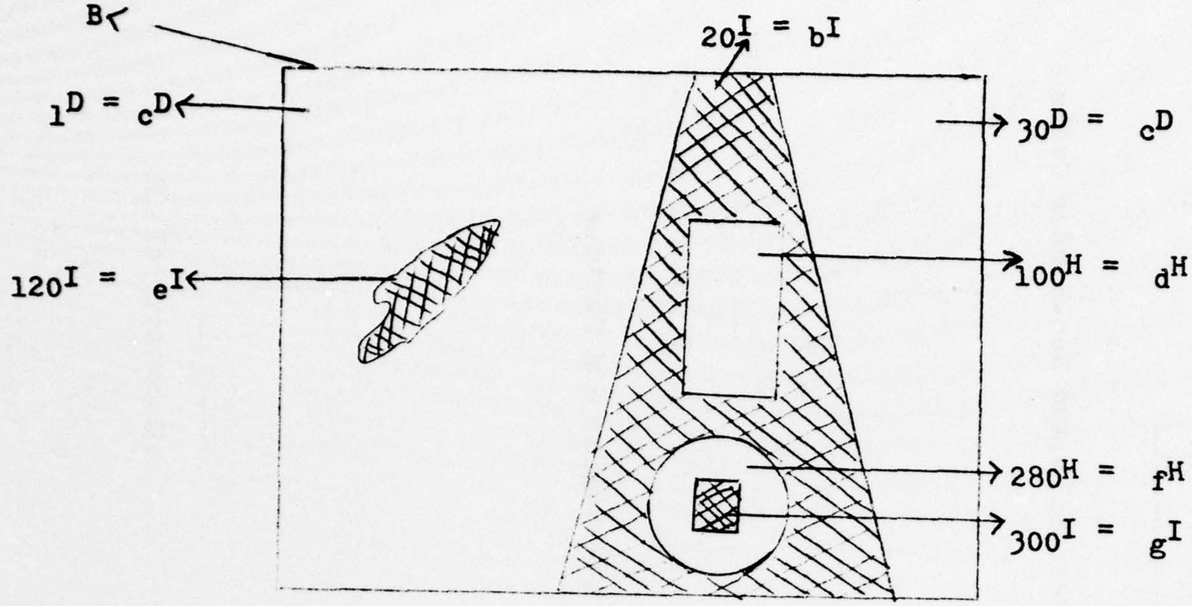


Figure 6. Picture and Background

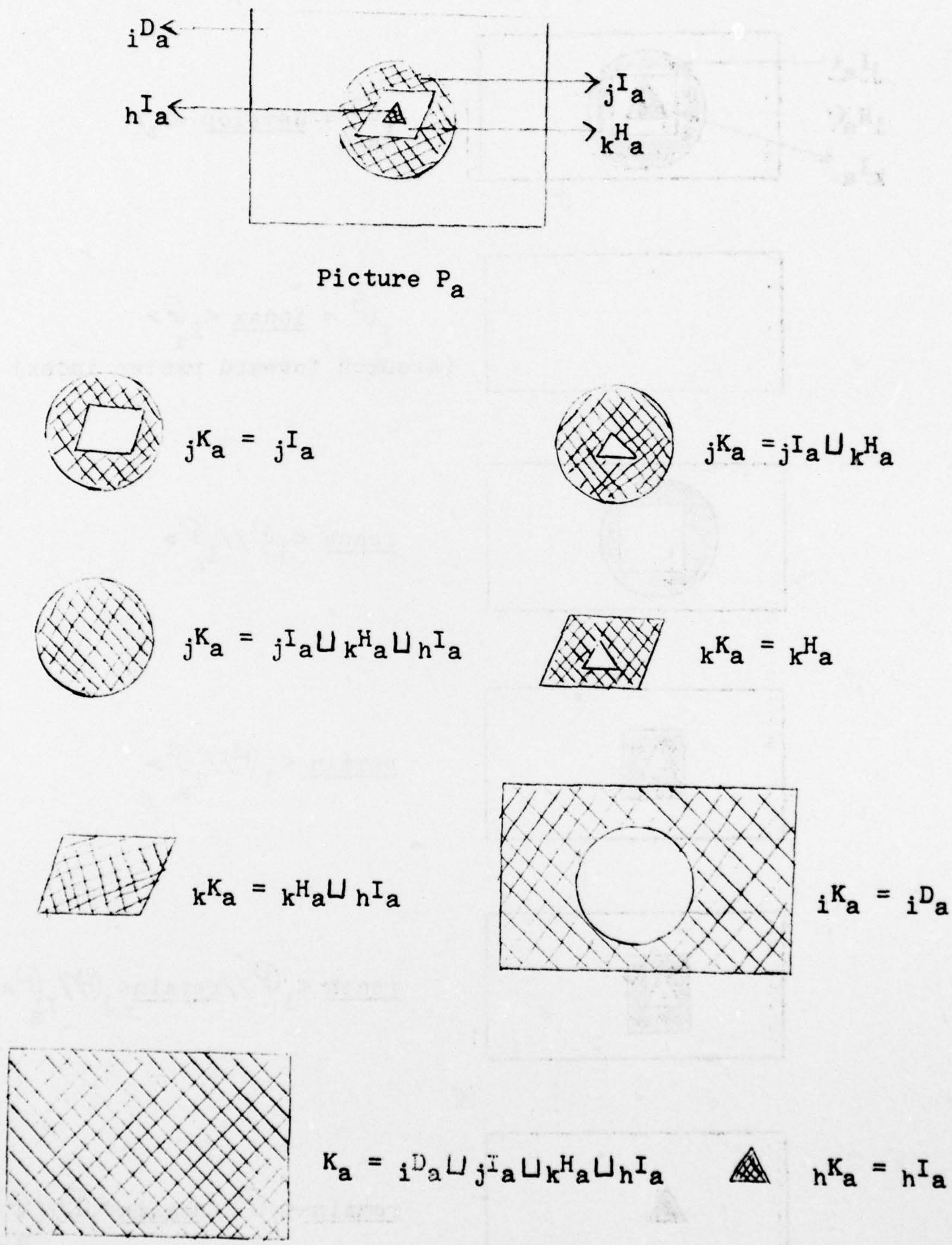
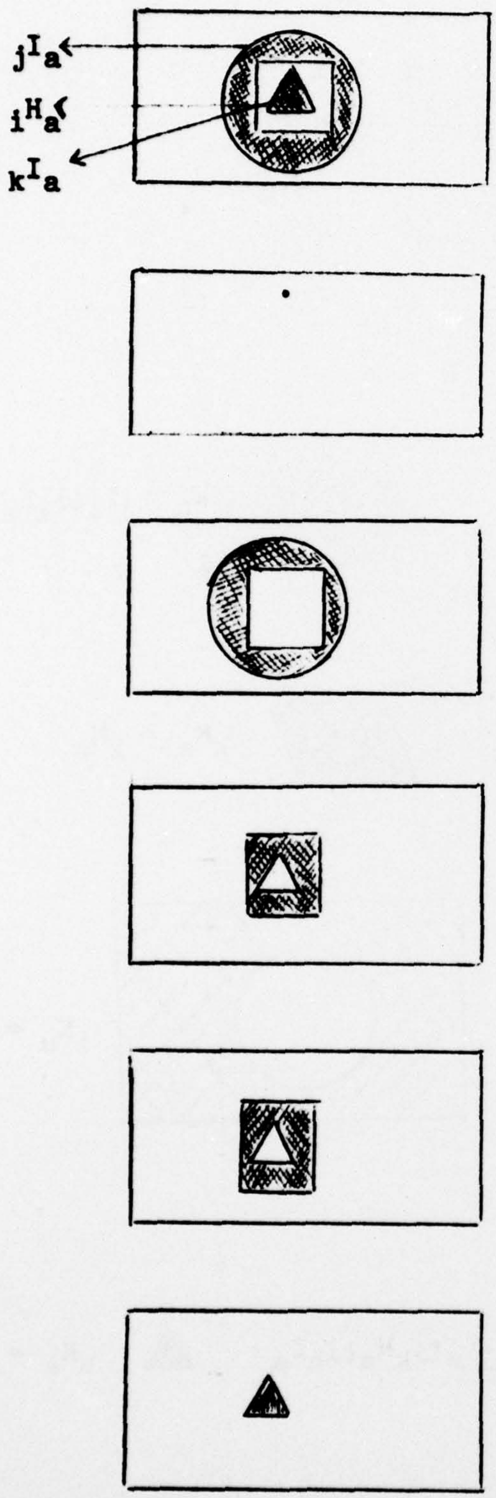


Figure 7. Some Possible Collages From Picture P_a



${}_{I_a}\mathcal{P}$ = <u>develop</u> $\langle P_a \rangle$

${}_{j}\mathcal{P}$ = <u>index</u> $\langle {}_{I_a}\mathcal{P} \rangle$
(assumes forward raster index)

<u>reach</u> $\langle {}_{j}\mathcal{P} // {}_{I_a}\mathcal{P} \rangle$

<u>remain</u> $\langle {}_{j}\mathcal{P} // {}_{I_a}\mathcal{P} \rangle$

<u>reach</u> $\langle {}_{i}\mathcal{P} //$ <u>remain</u> $\langle {}_{j}\mathcal{P} // {}_{I_a}\mathcal{P} \rangle$

<u>remain</u> $\langle {}_{i}\mathcal{P} //$ <u>remain</u> $\langle {}_{j}\mathcal{P} // {}_{I_a}\mathcal{P} \rangle$

Figure 8. Use of Print Operations

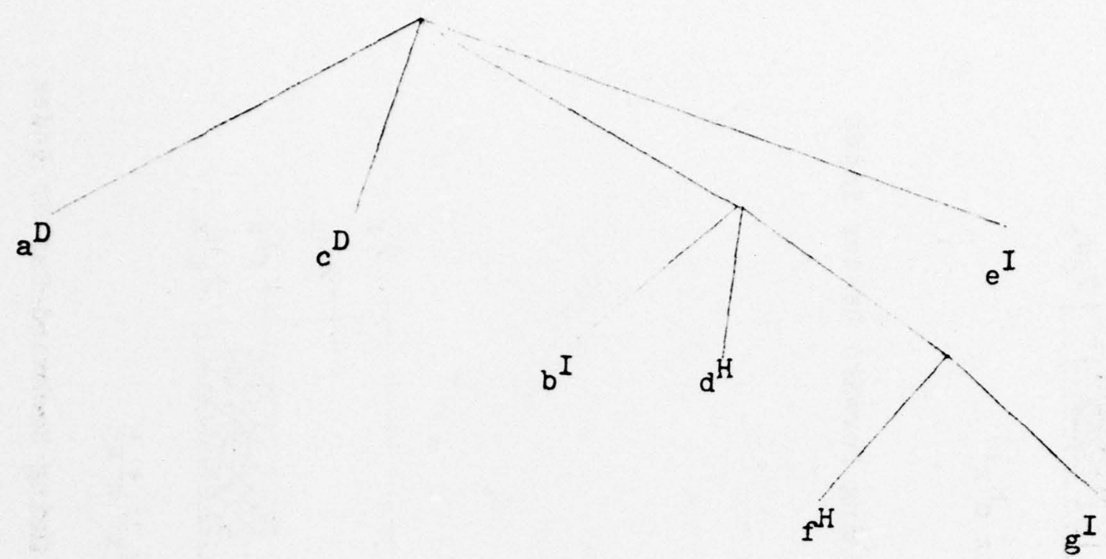
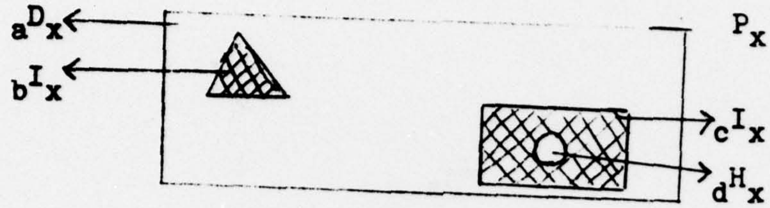
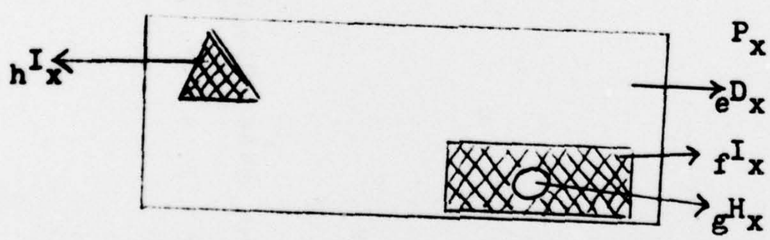


Figure 9. Hasse Diagram for String (9)



$(_aD_x\ _bI_x\ (_cI_x\ _dH_x))$

Figure 10. Decomposition Using Forward Raster Index



$(_eD_x\ (_fI_x\ _gH_x)\ _hI_x)$

Figure 11. Decomposition Using Backward Raster Index

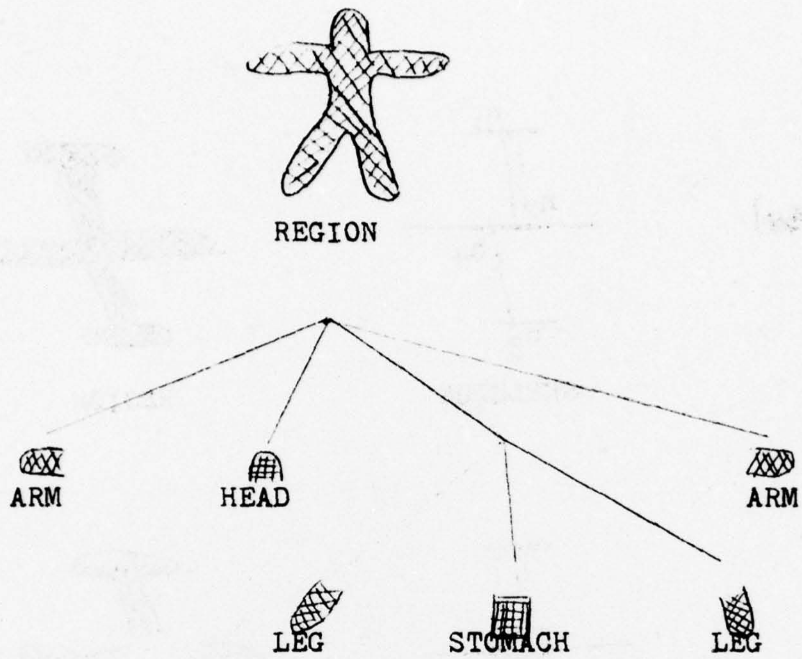


Figure 12. Decomposition of a Simply-Connected Region

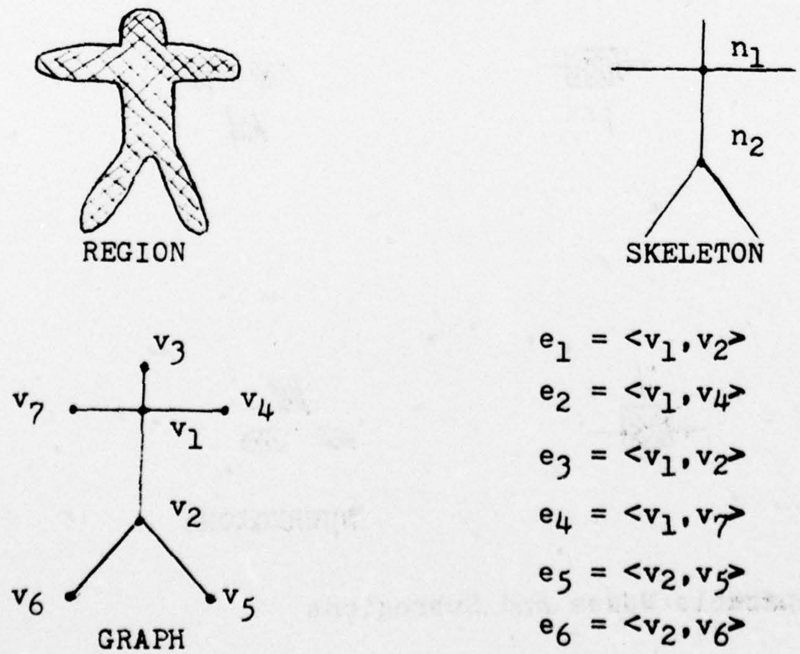


Figure 13. Skeleton and Graph of a Region

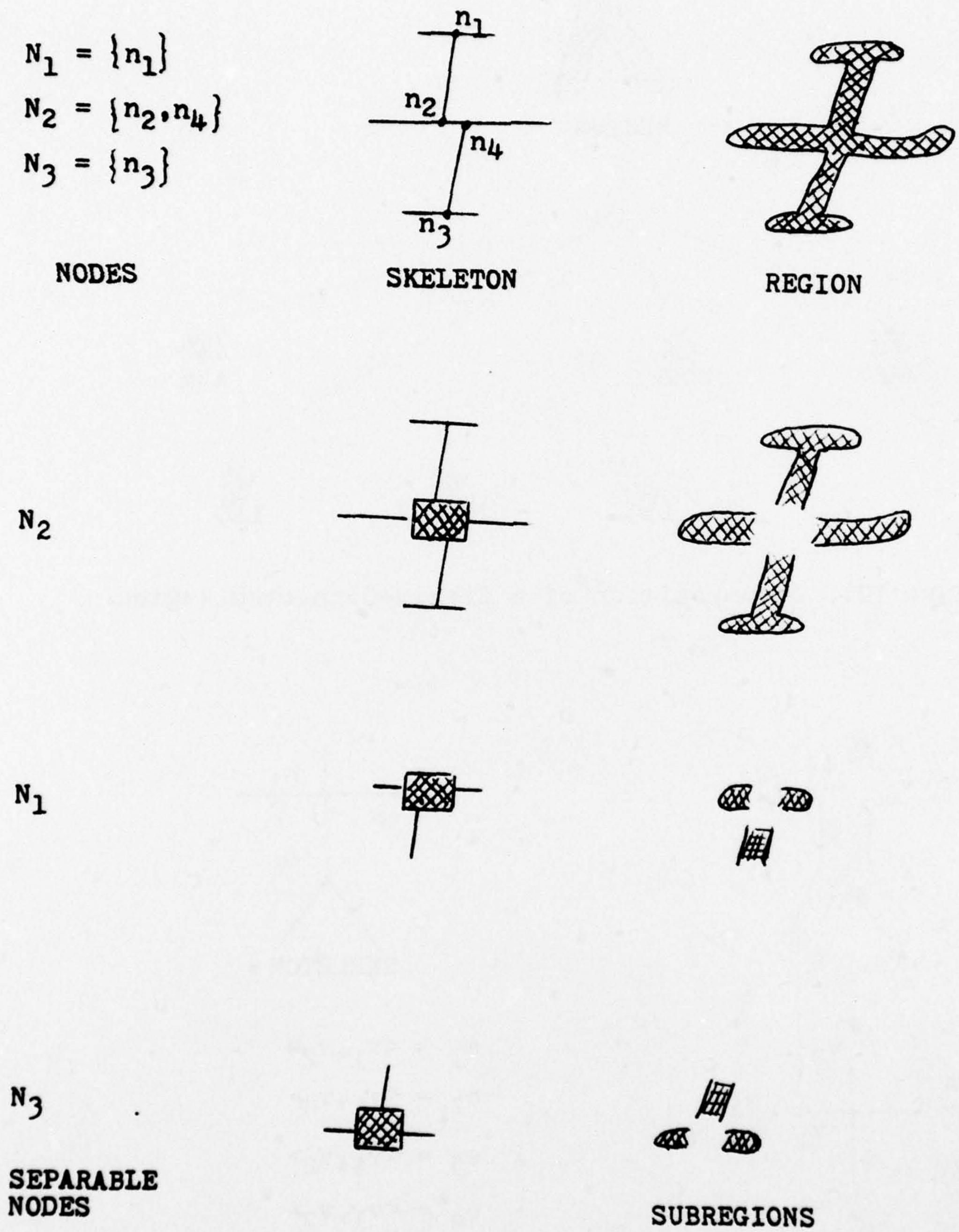


Figure 14. Separable Nodes and Subregions

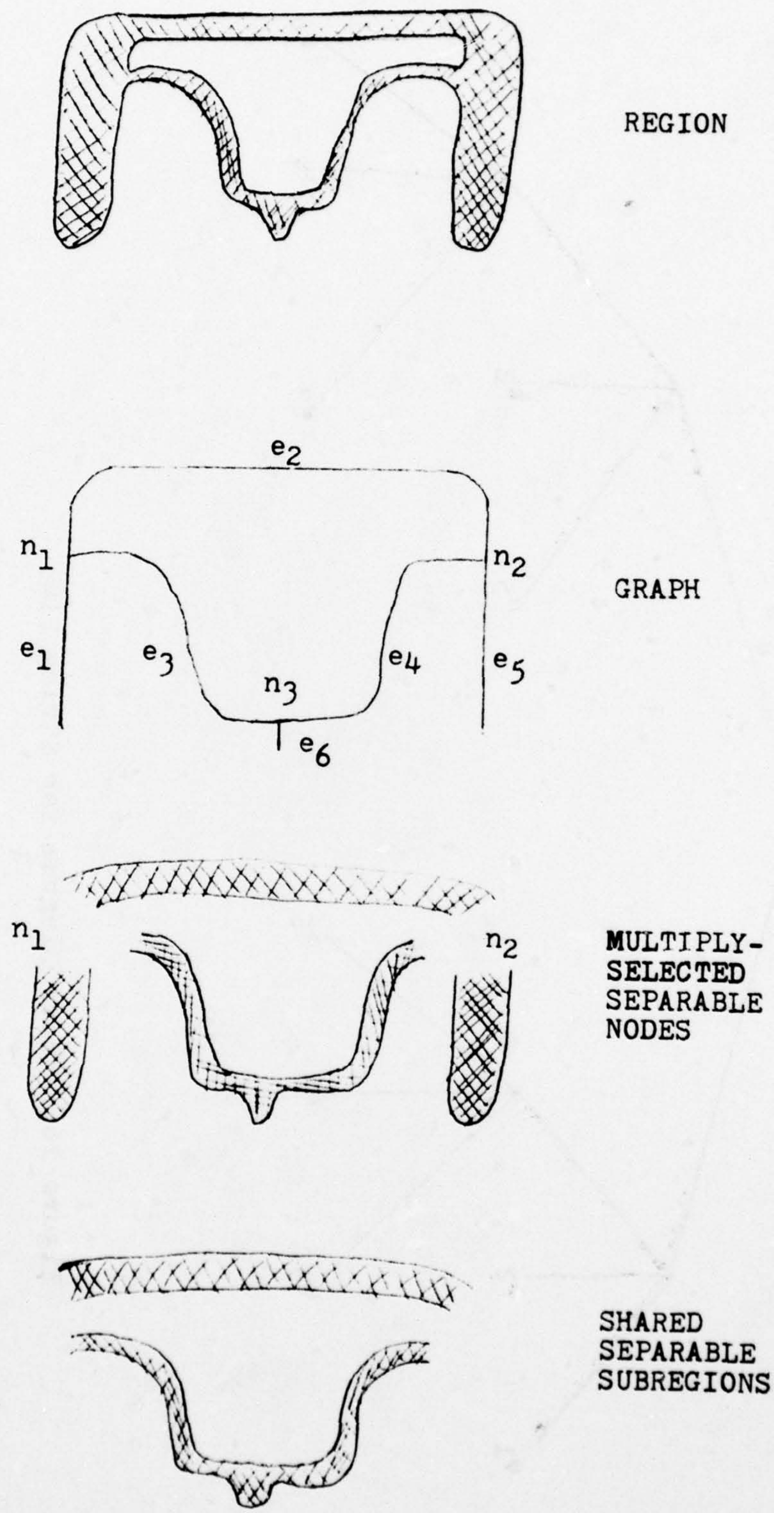


Figure 15. Shared Separable Subregions

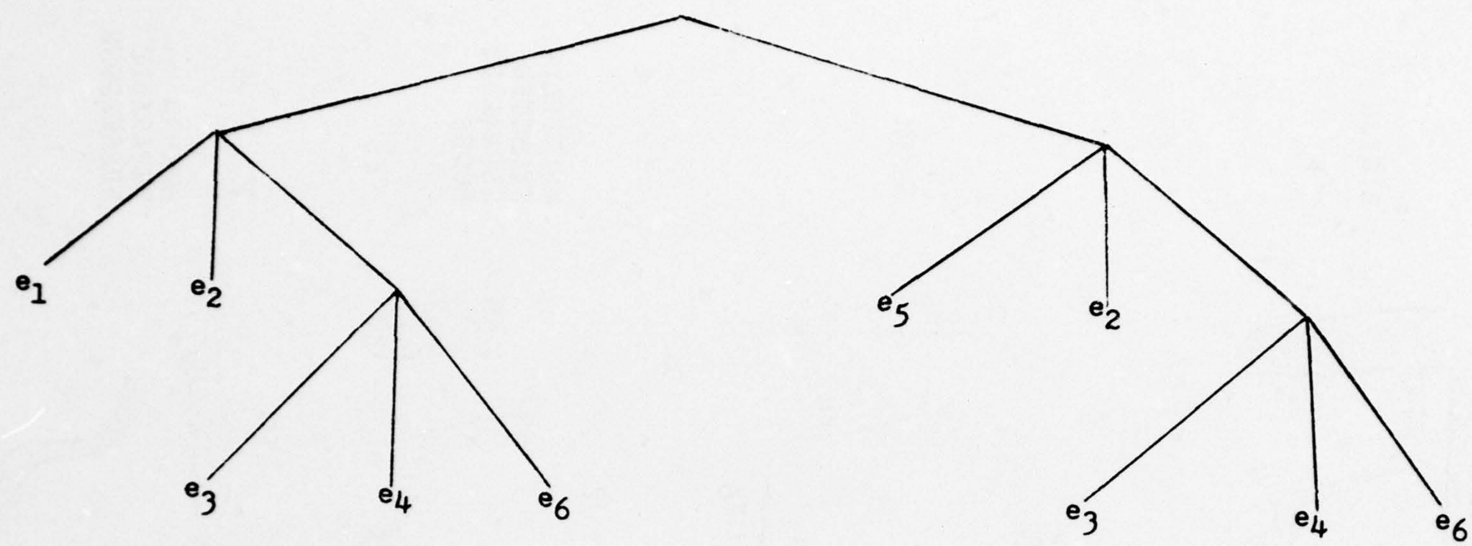


Figure 16. Hasse Diagram for String (10)

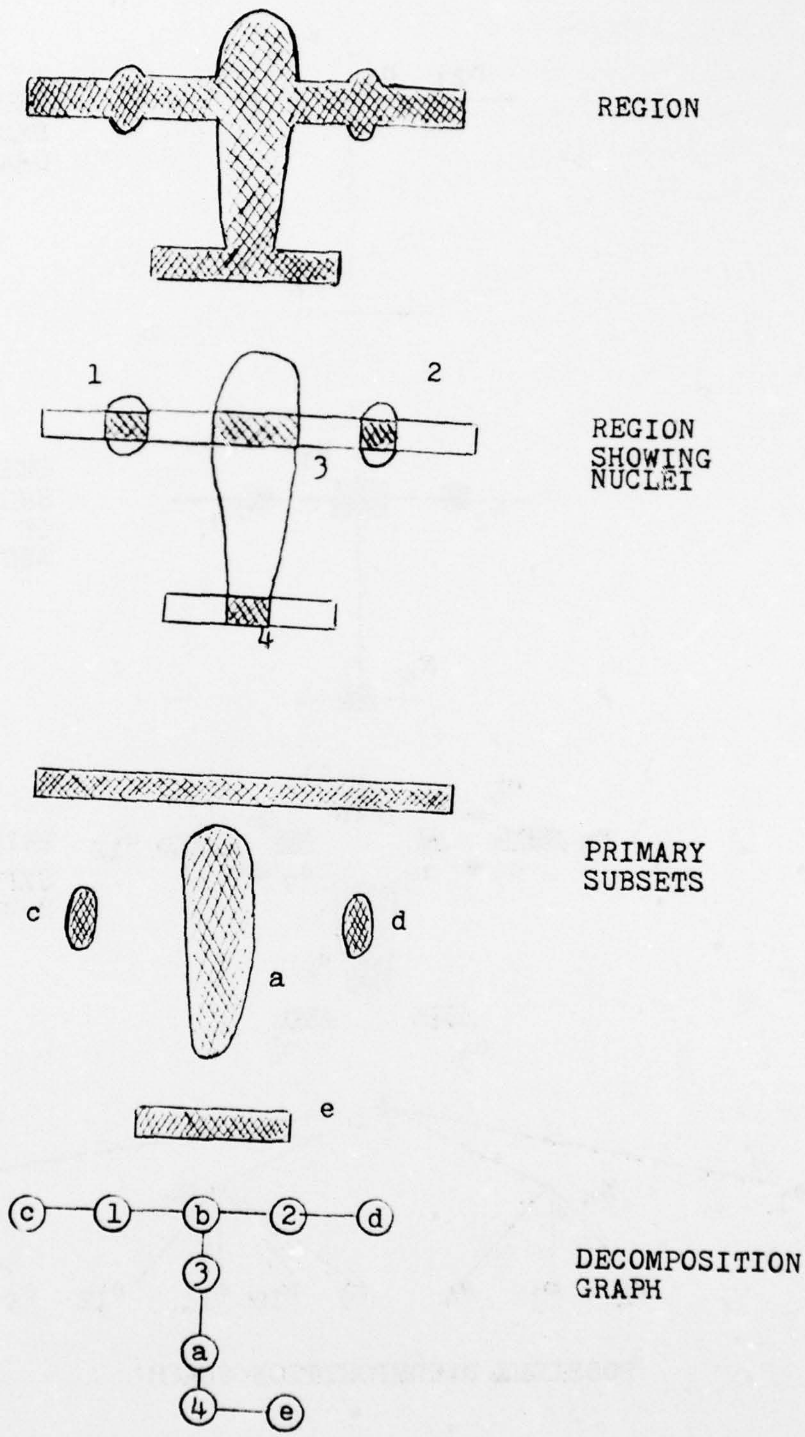


Figure 17. Decomposition Using Primary Subsets and Nuclei

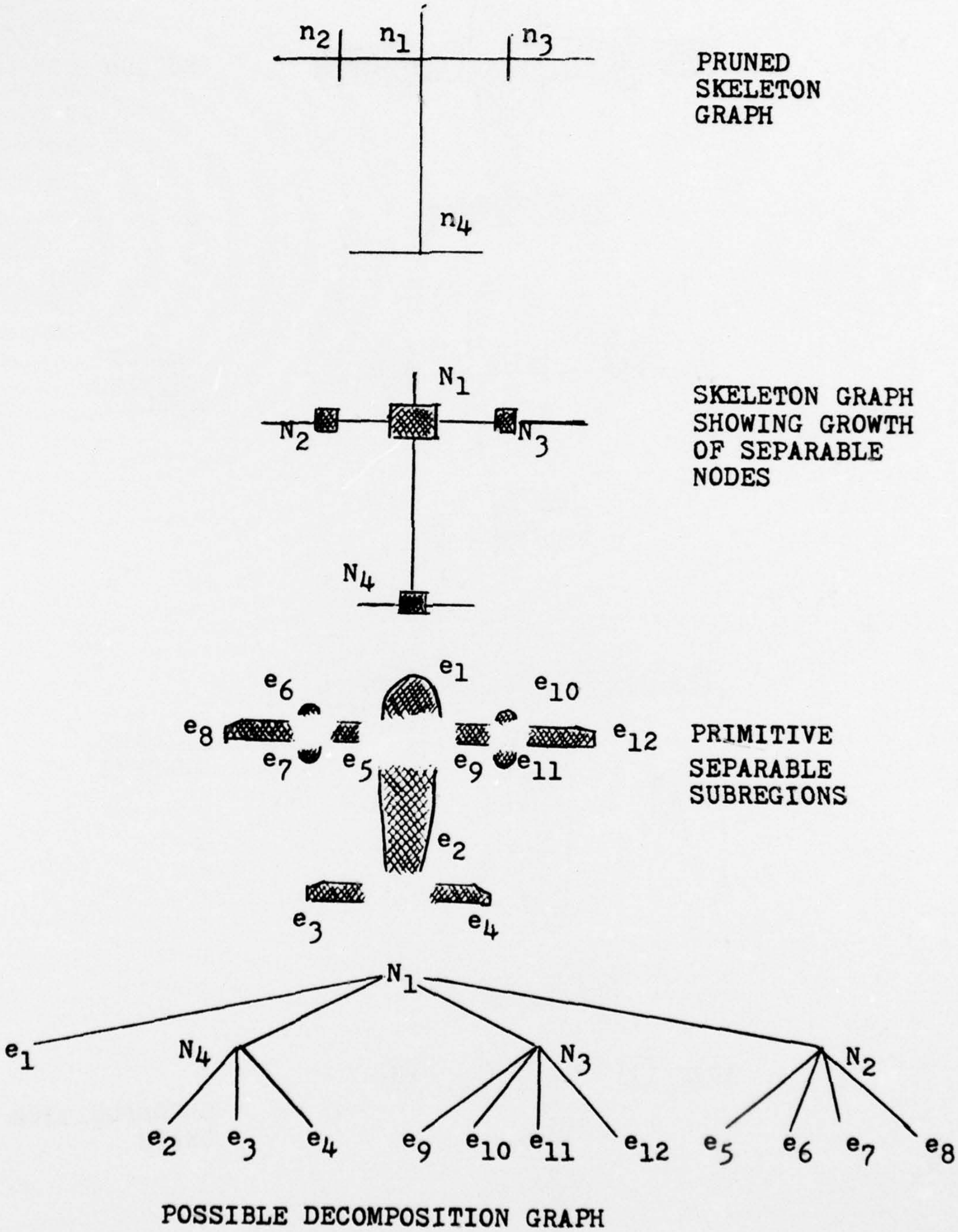


Figure 18. Decomposition Using Separable Subregions and Separable Nodes

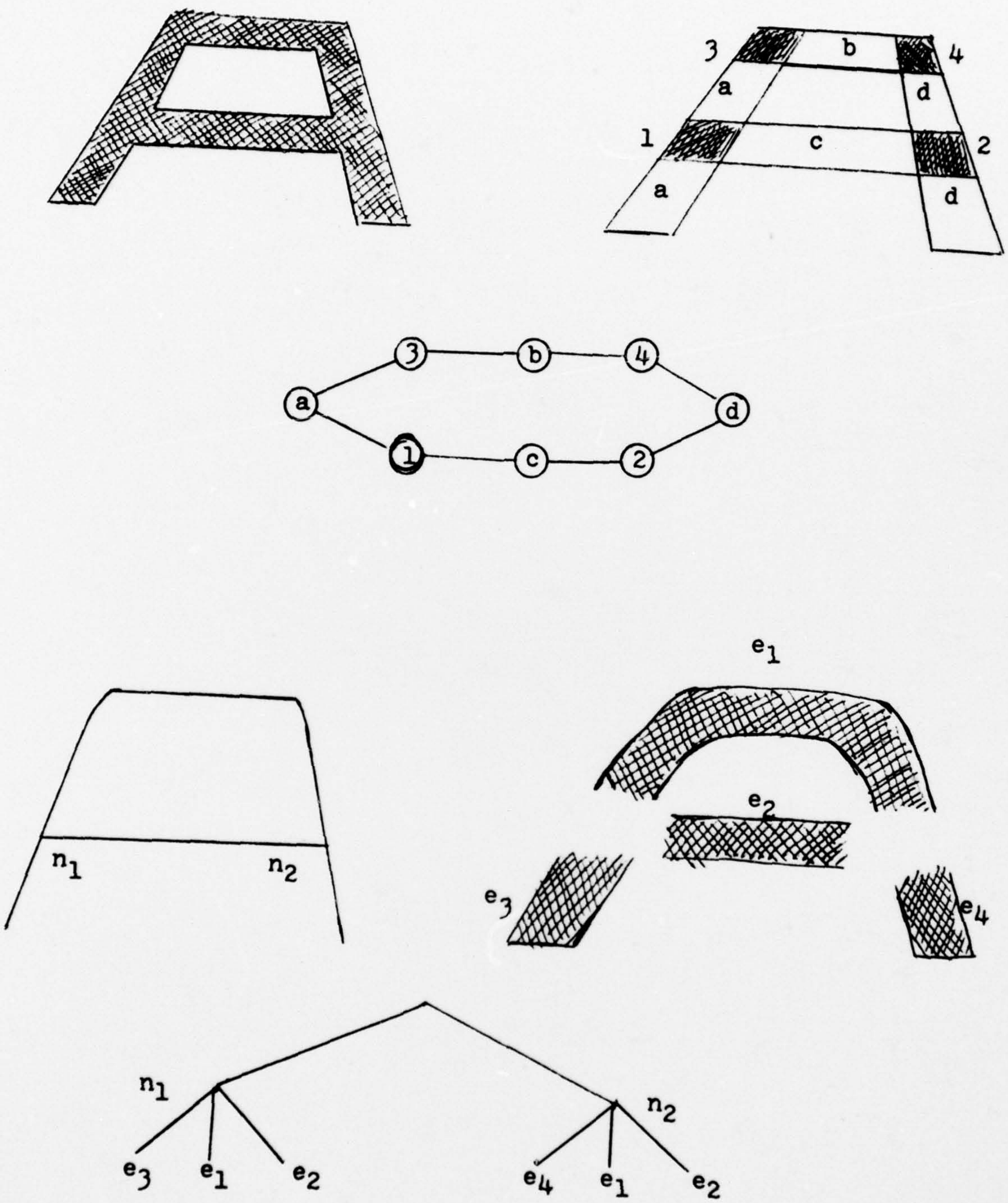


Figure 19. Comparing Nuclei and Separable Nodes